DP Analysis & Approaches (SL/HL)

Maths with lemon

Lesson 2.2 Systems of Two Equations

Mathematician: _____

Prior Learning: Number and Algebra	Guidance, clarification and syllabus links
Solving systems of linear equations in two variables.	

Warm Up

Determine whether the following points lie on the line

$$y = \frac{1}{2}x - \frac{5}{3}.$$

a) $\left(2, -\frac{2}{3}\right)$

b) $\left(5, \frac{7}{5}\right)$

c) $\left(\frac{3}{4}, -\frac{31}{24}\right)$

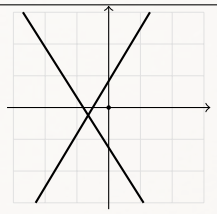
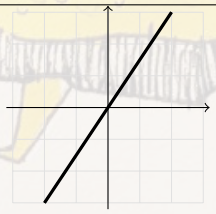
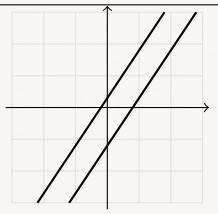
Solving by Substitution

Worked Example	You Try
Solve the system using the substitution method. $y = -7x$ $y = 9x + 32$	Solve the system using the substitution method. $y = 5x - 24$ $y = -3x$
Solve the system using the substitution method. $y = -x$ $10x + 2y = 40$	Solve the system using the substitution method. $8x + 10y = 8$ $x = -y$
Solve the system using the substitution method. $-x - 3 = y$ $-3x - 8y = 4$	Solve the system using the substitution method. $9x + y = 21$ $-x - 3 = y$

Solving Using Elimination

Worked Example	You Try
Solve the system using the elimination method. $3x + 2y = -16$ $-3x - 8y = 46$	Solve the system using the elimination method. $3x - 3y = 18$ $3x - 8y = -2$
Solve the system using the elimination method. $9x - 2y = -50$ $-3x + 7y = 4$	Solve the system using the elimination method. $8x + y = 17$ $4x + 3y = 31$
Solve the system using the elimination method. $x - 3y = -14$ $x - 7y = -26$	Solve the system using the elimination method. $4x + 3y = -19$ $3x + 3y = -15$

Types of Systems

Independent Consistent	Dependent Consistent	Inconsistent
One possible solution.	Infinite solutions.	No solutions.
One point of intersection between the lines.	Lines overlap creating infinite points of intersection between the lines.	Lines are parallel.
Lines have different slope.	Lines have same slope and same y-intercept.	Lines have same slope and different y-intercept.
		

Solving Dependent Systems

Worked Example	You Try
Solve the system using any method. $3x + 78 = 21y$ $x - 7y = -26$	Solve the system using any method. $-\frac{1}{2}x + \frac{3}{2}y - 7 = 0$ $x + 26 = 7y$

Solving Inconsistent Systems

Worked Example	You Try
Solve the system using any method. $3x + 21 = 21y$ $2x - 14y = -52$	Solve the system using any method. $6y = 16 + 2x$ $14y + 13 = 2x$

Finding Missing Coefficients

Worked Example	You Try
Consider the system below. For the system: **(a)** Find the solution in terms of a and b. **(b)** Determine the values of a and b for which (i) the system is independent (ii) the system is dependent (iii) the system is inconsistent. $y = ax - 24$ $y = -2x + b$	Consider the system below. For the system: **(c)** Find the solution in terms of a and b. **(d)** Determine the values of a and b for which (i) the system is independent (ii) the system is dependent (iii) the system is inconsistent. $y = 4x + b$ $y = ax$

Consider the system below. For the system:
(e) Find the solution in terms of a and b.
(f) Determine the values of a and b for which
(i) the system is independent
(ii) the system is dependent
(iii) the system is inconsistent.

$$-x + 9y = b$$
$$3x - ay = 9$$

Consider the system below. For the system:
(g) Find the solution in terms of a and b.
(h) Determine the values of a and b for which
(i) the system is independent
(ii) the system is dependent
(iii) the system is inconsistent.

$$7x + 14y = b$$
$$-2x - ay = -8$$

Consider the system below. For the system:
(i) Find the solution in terms of a and b.
(j) Determine the values of a and b for which
(i) the system is independent
(ii) the system is dependent
(iii) the system is inconsistent.

$$8x + by = -28$$
$$-4x = y + a$$

Consider the system below. For the system:
(k) Find the solution in terms of a and b.
(l) Determine the values of a and b for which
(i) the system is independent
(ii) the system is dependent
(iii) the system is inconsistent.

$$-x + 3y = b$$
$$ay = 2x$$

Solutions

Lesson 2.2 Systems of Two Equations

Warm Up

a) $\left(2, -\frac{2}{3}\right)$: **Yes**, since

$$\frac{1}{2}(2) - \frac{5}{3} = 1 - \frac{5}{3} = -\frac{2}{3}.$$

b) $\left(5, \frac{7}{5}\right)$: **No**, since

$$\frac{1}{2}(5) - \frac{5}{3} = \frac{5}{2} - \frac{5}{3} = \frac{5}{6} \neq \frac{7}{5}.$$

c) $\left(\frac{3}{4}, -\frac{31}{24}\right)$: **Yes**, since

$$\frac{1}{2}\left(\frac{3}{4}\right) - \frac{5}{3} = \frac{3}{8} - \frac{5}{3} = -\frac{31}{24}.$$

Solving by Substitution

Worked Example	You Try
$-7x = 9x + 32$ $-16x = 32 \Rightarrow x = -2$ $y = -7(-2) = 14$ $\boxed{(x, y) = (-2, 14)}$	$5x - 24 = -3x$ $8x = 24 \Rightarrow x = 3$ $y = -3(3) = -9$ $\boxed{(x, y) = (3, -9)}$
Since $y = -x$, $10x + 2(-x) = 40$ $8x = 40 \Rightarrow x = 5, \quad y = -5$ $\boxed{(x, y) = (5, -5)}$	Since $x = -y$, then $y = -x$: $8x + 10(-x) = 8$ $-2x = 8 \Rightarrow x = -4, \quad y = 4$ $\boxed{(x, y) = (-4, 4)}$
Since $y = -x - 3$, $-3x - 8(-x - 3) = 4$ $5x + 24 = 4 \Rightarrow x = -4$ $y = 4 - 3 = 1$ $\boxed{(x, y) = (-4, 1)}$	Since $y = -x - 3$, $9x - x - 3 = 21$ $8x = 24 \Rightarrow x = 3$ $y = -3 - 3 = -6$ $\boxed{(x, y) = (3, -6)}$

Solving Using Elimination

Worked Example	You Try
Add the equations: $(3x+2y)+(-3x-8y)=-16+46$ $-6y=30 \Rightarrow y=-5$ $3x+2(-5)=-16 \Rightarrow x=-2$ $\boxed{(-2,-5)}$	Subtract the equations: $(3x-8y)-(3x-3y)=-2-18$ $-5y=-20 \Rightarrow y=4$ $3x-12=18 \Rightarrow x=10$ $\boxed{(10,4)}$
Multiply the second equation by 3: $-9x+21y=12$ Add: $19y=-38 \Rightarrow y=-2$ $9x+4=-50 \Rightarrow x=-6$ $\boxed{(-6,-2)}$	From $8x+y=17$, $y=17-8x.$ Substitute: $4x+3(17-8x)=31$ $-20x=-20 \Rightarrow x=1, \quad y=9$ $\boxed{(1,9)}$
Subtract: $(x-7y)-(x-3y)=-26-(-14)$ $-4y=-12 \Rightarrow y=3$ $x-9=-14 \Rightarrow x=-5$ $\boxed{(-5,3)}$	Subtract: $(4x+3y)-(3x+3y)=-19-(-15)$ $x=-4$ $-12+3y=-15 \Rightarrow y=-1$ $\boxed{(-4,-1)}$

Dependent and Inconsistent Systems

Dependent worked example

$$3x+78=21y \quad \Longrightarrow \quad x+26=7y$$

and

$$x-7y=-26 \quad \Longrightarrow \quad x+26=7y.$$

The equations represent the same line, so there are

$\boxed{\text{infinitely many solutions}}$.

Dependent "You Try" as written in the worksheet

The equations are

$$-\frac{1}{2}x+\frac{3}{2}y-7=0 \quad \Longrightarrow \quad x-3y=-14,$$

and

$$x-7y=-26.$$

Subtracting gives $4y=12$, so $y=3$ and $x=-5$:

$$\boxed{(x,y)=(-5,3)}.$$

Note: as written, this system is independent rather than dependent.

Inconsistent worked example

From

$$3x + 21 = 21y$$

we obtain

$$x - 7y = -7.$$

The second equation gives

$$x - 7y = -26.$$

This is a contradiction, so

$$\boxed{\text{no solution}}.$$

Inconsistent "You Try" as written in the worksheet

$$6y = 16 + 2x \quad \Longrightarrow \quad x - 3y = -8,$$
$$14y + 13 = 2x \quad \Longrightarrow \quad x - 7y = \frac{13}{2}.$$

Solving gives

$$y = -\frac{29}{8}, \qquad x = -\frac{151}{8}.$$

Therefore

$$\boxed{\left(-\frac{151}{8}, -\frac{29}{8}\right)}.$$

Note: as written, this system is consistent rather than inconsistent.

Finding Missing Coefficients — Solutions

(a)–(b)

$$y = ax - 24, \qquad y = -2x + b.$$

For $a \neq -2$,

$$ax - 24 = -2x + b$$

so

$$x = \frac{b + 24}{a + 2}, \qquad y = \frac{ab - 48}{a + 2}.$$

Hence:

$$\boxed{\text{Independent: } a \neq -2}$$

$$\boxed{\text{Dependent: } a = -2,\ b = -24}$$

$$\boxed{\text{Inconsistent: } a = -2,\ b \neq -24}$$

(c)–(d)

$$y = 4x + b, \qquad y = ax.$$

For $a \neq 4$,

$$4x + b = ax$$

so

$$x = \frac{b}{a - 4}, \qquad y = \frac{ab}{a - 4}.$$

Hence:

$$\boxed{\text{Independent: } a \neq 4}$$

$$\boxed{\text{Dependent: } a = 4,\ b = 0}$$

$$\boxed{\text{Inconsistent: } a = 4,\ b \neq 0}$$

(e)–(f)

$$-x + 9y = b, \qquad 3x - ay = 9.$$

For $a \neq 27$,

$$y = -\frac{3(b + 3)}{a - 27}, \qquad x = -\frac{ab + 81}{a - 27}.$$

Hence:

$$\boxed{\text{Independent: } a \neq 27}$$

$$\boxed{\text{Dependent: } a = 27,\ b = -3}$$

$$\boxed{\text{Inconsistent: } a = 27,\ b \neq -3}$$

(g)–(h)

$$7x + 14y = b, \qquad -2x - ay = -8.$$

For $a \neq 4$,

$$x = \frac{112 - ab}{7(4 - a)}, \qquad y = \frac{2(b - 28)}{7(4 - a)}.$$

Hence:

$$\boxed{\text{Independent: } a \neq 4}$$

$$\boxed{\text{Dependent: } a = 4,\ b = 28}$$

$$\boxed{\text{Inconsistent: } a = 4,\ b \neq 28}$$

Finding Missing Coefficients — Solutions Continued

(i)–(j)

$$8x + by = -28, \qquad -4x = y + a.$$

For $b \neq 2$,

$$x = \frac{28 - ab}{4(b-2)}, \qquad y = \frac{2(a-14)}{b-2}.$$

Hence:

$$\boxed{\text{Independent: } b \neq 2}$$

$$\boxed{\text{Dependent: } b = 2,\ a = 14}$$

$$\boxed{\text{Inconsistent: } b = 2,\ a \neq 14}$$

(k)–(l)

$$-x + 3y = b, \qquad ay = 2x.$$

For $a \neq 6$,

$$x = \frac{ab}{6-a}, \qquad y = \frac{2b}{6-a}.$$

Hence:

$$\boxed{\text{Independent: } a \neq 6}$$

$$\boxed{\text{Dependent: } a = 6,\ b = 0}$$

$$\boxed{\text{Inconsistent: } a = 6,\ b \neq 0}$$

Summary of special cases

System	Dependent	Inconsistent
(a)–(b)	$a = -2,\ b = -24$	$a = -2,\ b \neq -24$
(c)–(d)	$a = 4,\ b = 0$	$a = 4,\ b \neq 0$
(e)–(f)	$a = 27,\ b = -3$	$a = 27,\ b \neq -3$
(g)–(h)	$a = 4,\ b = 28$	$a = 4,\ b \neq 28$
(i)–(j)	$b = 2,\ a = 14$	$b = 2,\ a \neq 14$
(k)–(l)	$a = 6,\ b = 0$	$a = 6,\ b \neq 0$

Maths with Lemon

Mathematics made clear.